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# Residual Stresses Field Estimation Based on Deformation Force Data Using Gaussian Process Latent Variable Model

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## Abstract

Residual stresses field inside the bulk material is the main factor of the part deformation, which is a vital issue for large-scaled monolithic components manufacturing in aircraft industry. The estimation of residual stresses is the base of the part deformation control. Non-uniform residual stresses field is distributed among the whole part, and differs from part to part. However, existing residual stress measurement's methods can only measure the sample's residual stress and the detection depths are limited by physical principle, which makes it hard to measure the heavy thickness part. In order to address this issue, this paper presents a method of estimating part residual stresses fields and their uncertainty in machining process based on a novel Bayesian statistical model integrated with observation data (deformation force). Deformation forces data can be easily and accurately monitored during machining processing via several fixture devices, and contain information that can be related to the residual stress. In order to solve the problem of how to infer an unobservable residual stresses field by using sparse observed deformation force data, this work introduces a Bayesian framework. The unmeasurable and unobservable residual stresses field is deemed as the latent physics field and the Gaussian process is specified for the latent field as a prior with parameters in kernel function to reduce the solution space and also introduces proper Distribution hypothesis on deformation force data. The model provides an acceptable estimation result with the limited observation data. The proposed method provides an effective way to estimate interior residual stress field in the machining processing of monolithic components part in terms of probability, and will help the selection of deformation control strategy.

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**Keywords:** Residual stress; deformation force; Bayesian framework.

## 1. Introduction

Residual stresses are mechanical stresses remaining in bulk material, which is at self-equilibrium and satisfy the static force and moment equilibrium conditions [1,2]. Residual stresses are caused by uneven volume changes in the internal of the metal due to misfits between different regions or sharp thermal gradients. These are related to the material's processing history. Aluminum alloy raw materials that are widely used in aviation

manufacturing exhibit residual stress profiles and can cause unexpected non-conformance parts, rework cost and delays.

Machining deformation caused by residual stresses is a significant issue during the machining of thin-walled parts with low rigidity [3]. In the manufacturing of aircraft structural parts, it is necessary to mill large pieces of bulk material with residual stresses into thin-walled structural parts, and the material removed rate can be up to 95%. The original balance of residual stresses is disrupted during the machining process and the

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residual stresses are redistributed when releasing fixtures, which causes serious deformation of the structure parts. In this process, the initial residual stresses are the main cause of the machining deformation. So, initial residual stresses are the initial condition for many deformation control and prediction models, such as deformation control based on finite element simulation [4], and it is very important for predicting deformation and guiding deformation control. However, the initial residual stresses measurement is still a challenge, especially in manufacturing due to the thickness of blank.

In the present paper a Bayesian model is developed for the estimation of the residual stresses profile. The model provides an acceptable estimation result with the limited observation data. The remaining of the paper is structured into a literature review section, followed by the developed methodology, a numerical example that validates the methodology, and the key conclusions.

## 2. Literature review

The residual stress measurement methods can be divided into destructive and non-destructive ones.

Destructive testing methods are based in removing layers of material and then measuring the resulting deformations (displacement or strain) in the adjacent material to calculate the residual stresses. Commonly used techniques include Sachs' method, layer removal method, hole-drilling method, and slitting (crack compliance) method [5].

Non-destructive measurement methods are mostly physical methods, including X-ray diffraction method, synchrotron radiation method, neutron diffraction method, magnetic method and ultrasonic method, etc. [2] At present, the most commonly used nondestructive measurement method for residual stress is X-ray diffraction. Based on Bragg's law of diffraction, it can measure the surface stress within 5-20 $\mu\text{m}$  of the surface. The basic principle of neutron diffraction measurement of residual stress is the same as that of x-ray diffraction measurement of stress. For most engineering materials, the penetrating power of neutron diffraction is on the order of centimeters and it can measure the three-dimensional stress inside the material. However, it takes longer to test, it has a higher cost, and it cannot be used in real time of the manufacturing process [6]. Although the ultrasonic method and the magnetic strain method can measure the internal stress of the material, their accuracy is greatly affected by the microstructure. Most of the non-destructive testing methods can only detect residual stresses at the surface or near-surface layers. Experimental techniques for directly testing residual stresses of arbitrary internal points are lacking.

Based on traditional detection technology, researchers have made corresponding expanded researches to meet the requirements of their respective fields. Some researchers simulate the actual manufacturing processes for mechanical parts, such as casting, machining and welding, to reconstruct residual stress fields [7]. Other researchers pay more attention to calculate the residual stress from the measured data of the finished product. Fengyun Wang [8] proposed a finite element implementation technique to determine the surface residual stress – body residual stress function, so that the complete

residual stresses state of a solid may be predicted from surface stress measurement data. Hjelm et al. [9] originally used natural input modal analysis in the determination of stress histories. Schajerz and Prime [10] proposed a generic inverse solution approach for destructive residual stress measurements. Hatamleh et al. [11] quantified the uncertainty caused by X-rays by introducing a joint probability density function, which improves the credibility of the data.

Initial residual stresses are in the micro scale and are affected by various factors. The traditional detection technologies mentioned above require the braking of the materials totally and often use samples to measure residual stresses to represent the original material residual stress field. However, in the real environment, initial residual stress distribution is different from each other, even if they come from the same batch, especially the die forging stock. The existing residual stress measurements can only measure a replaced residual stress field and need to destroy the whole material, which is not accurate and hysteretic, so it can not satisfy the requirements of the deformation prediction and control.

In order to solve the problem mentioned above, this paper proposes a method to estimate the residual stresses using limited deformation forces, and provides the uncertainty of the residual stresses field, in which sparse observation data deformation force monitored on-line is used to estimate the unobservable field residual stress.

## 3. Methodology

Although residual stresses are difficult to measure directly, in manufacturing process of parts, there are specific measurable physical quantities, which represent the influence of residual stresses on deformation. It is an effective way to estimate residual stresses by those quantities. In this paper, an initial residual stress field estimation method based on Bayesian framework using deformation force is proposed. In this method, the residual stresses field is seen as a latent variable field, which is fixed during machining. And the deformation force, which can be obtained during manufacturing process, is deemed as the observable data and introduced to estimate the residual stress. It is easy to obtain the deformation force during the machining process through the fixture device.

### 3.1. The definition and acquisition of deformation force

This section will introduce the acquisition of deformation force in manufacturing process. As shown in Fig. 1, fixture is one of the necessary equipment in the production of aircraft structural parts, it plays the role of positioning and clamping. When the workpiece is released, it tends to deform due to the redistributed residual stress. When the deformation is limited by the fixture, it will impose a force on it, which is reaction force, as illustrated in Fig. 1. In this paper, the deformation force refers to the reaction force imposed on the fixtures.

There are currently some fixtures that can monitor the reaction force [12]. Therefore, the deformation force during processing can be easily measured. Compared to residual

stresses, deformation force is a quantity that can be accurately obtained.

$$\text{force}_{ij} = \text{function}(G_j, \sigma_0, \text{Coor}_i) \quad (1)$$

Where  $\text{force}_{ij}$  is the deformation force,  $G$  is the geometry,  $\sigma_0$  is initial residual stress field, and  $\text{Coor}_i$  is the position of the measurement device of deformation  $\text{force}_{ij}$ .

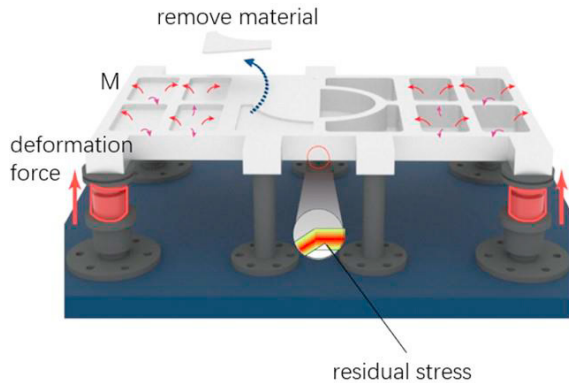


Fig. 1. Fixtures in aviation structural parts manufacturing.

During the machining process, the part geometry  $G$  is changing in the light of the machining planning. The deformation force changes only with the change in the part's geometry. As the part is fixed on the workbench, no deformation happens and thus the initial residual stresses are also fixed within the material. Therefore, fixed residual stresses can be estimated through the monitoring of the deformation forces.

### 3.2. Inferring process of residual stress field

The deformation force monitored during machining process is taken as the observation data, and as it is described in equation (1), it is a function of the residual stresses. So, the prediction of residual stresses is transformed into the process that using observable deformation force to infer non-observable residual stress. The key point to estimate latent fields by using sparse observations is the representation of the uncertainty. In this paper, the deformation force is deemed as exact, latent residual stresses field is deemed as uncertainty. Bayesian framework provides a good solution to the current problem. It is suitable for solving inverse probability problems based on prior knowledge and observation data, and only needs a small amount of data. In Bayesian framework, the uncertain field is a random field, and the Gaussian process is used to describe this field as a prior,

$$\sigma_0 \sim \text{GP}(\tilde{\sigma}_0, K) \quad (2)$$

where  $K$  is the covariance with parameters,  $\sigma_0$  is residual stress field, and  $\tilde{\sigma}_0$  is the baseline solution.

The deformation force follows the Gaussian distribution as described in the following equation:

$$\text{force}_{ij} \sim N(\text{function}(G_j, \sigma_0, \text{Coor}_i), \delta) \quad (3)$$

The process of estimating the residual stresses field by observed deformation force is deemed as the solving process of the posteriori distribution  $P(\text{Parameters} | \text{force})$ , in which the parameters are the whole parameters of the estimation process. The posterior distribution obtained from Bayesian formula will become very complicated and difficult to calculate. Method called Markov Chain Monte Carlo (MCMC) is used to solve the posterior probability. Monte Carlo method is to construct a probability model of a certain problem, conduct random and independent sampling, and use statistical methods to obtain the numerical characteristics of the model. This allows the algorithm to narrow in on the quantity that is being approximated from the distribution, even with a large number of random variables.

### 4. A numerical example

In order to validate the proposed method, a two-dimensional numerical part with residual stresses field is introduced here in simulation environment. Residual stresses level in workpiece is highly related to depth and weakly related to length and width[13]. The workpiece is divided into 10 layers in the depth direction and each layer is assigned a residual stress value. The residual stress field could be deemed as a function of the depth of the blank.

In order to obtain deformation force, finite element method (FEM) is used to calculate the deformation force with the initial condition, including residual stress, geometry and the position of the deformation force. The deformation force can be calculated from the simulation process, as illustrated in Fig. 2. The True generating function 'f' is the residual stress function, and observed data is the measured deformation force. According to Equation (1), if the residual stress field, bulk material properties and geometric properties are known, the deformation force can be obtained through FEM.

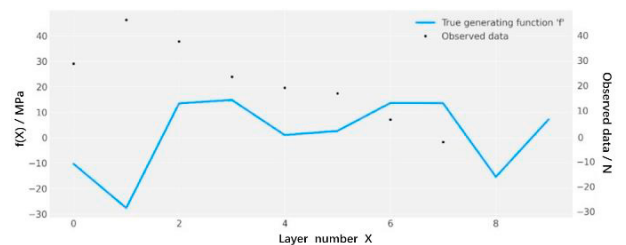


Fig. 2. Generated data.

Then the realization form and a priori of Gaussian process are defined.

As a quantity that is difficult to observe directly, the residual stress field is treated as a set of latent variables here, and the Gaussian process is used to be the prior of the variables.

Given a mean and covariance function, the residual stresses function is modeled as

$$f(x) \sim \text{GP}(m(x), k(x, x')) \quad (4)$$

where the  $m(x)$  and  $k(x, x')$  are the mean and covariance function respectively,  $\ell$  and  $\eta$  are parameters of the covariance function  $k(x, x')$ .

The length and amplitude of the kernel of Gaussian process  $\ell$  and  $\eta$  respectively obey Gamma distribution and Half Cauchy distribution. The observation data-deformation force obeys normal-distribution.

$$F \sim N(\mu, \sigma) \quad (5)$$

where the  $\sigma$  is the measurement error noise,  $\mu$  is the mean value of the deformation force.

To estimate the residual stress function, the posterior  $P(\text{Parameters} | F)$  needs to be calculated. However, it is difficult to solve it directly, so the MCMC method is used to estimate the distribution. After 3000 iterations, the implicit function posterior distribution is generated.

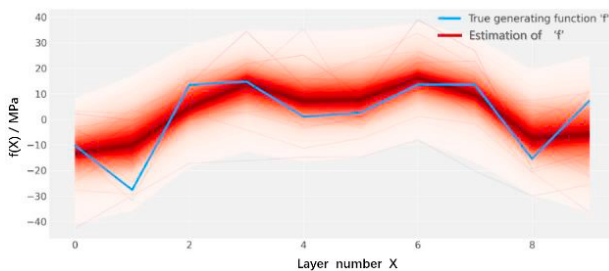


Fig. 3. posterior distribution over  $f(x)$  at the observed values.

As the calculated result shown in Fig. 3, the distribution of posterior probability we obtained is consistent with the true value of Fig. 2. The Root Mean Square Error of this estimation is 13.97. And the uncertainty of the residual stress value is properly covered the residual stress distributions. The parameters distribution estimated from the calculation are shown in Fig. 4, the tends to be stable and are close to true values. The figure illustrates that the result of the numerical case is acceptable. The method is preliminarily proved to be theoretically feasible.

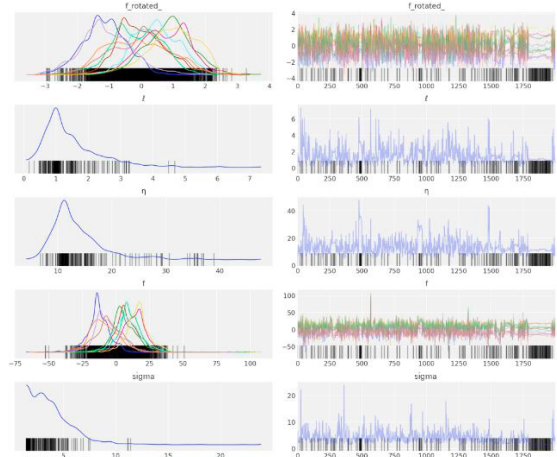


Fig. 4. Posterior distribution histogram and posterior sampling value of parameters.

## 5. Discussion and conclusion

In this paper, the problem of measurement of residual stress field is transformed into the estimation of residual stress by observable deformation force using Gaussian Process Latent variable model. And a numerical case is used to verify the proposed method. It provides a useful way to estimate the initial residual stress at any depth of parts in machining process with limited measured deformation force. It calculates the uncertainty of the residual stress distribution, which is import for on-line machining deformation control.

This work only validated in a two-dimensional model with one direction residual stress distribution. More complex situations need to be analyzed. For example, the residual stress is a three-dimensional filed and is not an average value in the thickness direction of the blank, which should be taken into account.

## 6. Future work

It should be noted that this study has been primarily concerned with the feasibility of a new algorithm for residual stress. In the further, more work is going to be done. In this work, a Bayes-based estimation method for the initial residual stress of aircraft structural parts is introduced. In this method, the prior of the residual stress is important in estimation result. A more useful prior will help to solve the problem more accurately with few deformation force data. So, in the next work, the knowledge of the residual stress mechanism will conclude and be added to the design of the covariance to form an effective prior on Gaussian process. Specifically, in Gaussian process regression, the covariance can be designed to be more consistent with the mechanism knowledge, making the prior model more reasonable, which is also the core advantage of the Bayesian method. In addition, experimental verification is also needed. Substituting the actual residual stress measurement and deformation force monitoring data can show the effectiveness of the method in this paper.

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